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1994 J. Phys.: Condens. Matter 6 3479

(http://iopscience.iop.org/0953-8984/6/19/003)

J. Phys.: Condens. Matter 6 (1994) 3479–3485. Printed in the UK

The effect of a high-frequency electric field on hypersound amplification in a superlattice

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Received 9 July 1993, in final form 1 February 1994

Abstract. Propagation of hypersound in a semiconductor superlattice in the presence of an external electric field of the form $E = E_0 + E_1 \cos(\omega t)$ has been studied theoretically. The analytical expressions obtained for the absorption coefficient are presented graphically. It is observed that in the absence of an AC electric field the dependence of Γ on E_0 is non-linear unlike the homogeneous bulk material where the dependence of Γ on E_0 is linear. The inclusion of an AC electric field affects the relation between Γ and E_0 depending on the values of E_1 and $\omega\tau$. For a given E_1, the peaks of the curve of Γ against E_0 decrease, shift and eventually oscillate for increasing values of $\omega\tau$, i.e. from $\omega\tau \ll 1$ to $\omega\tau \gg 1$.

1. Introduction

The study of acoustic effects in semiconductors and particularly of acoustic wave amplification by drift carriers has attracted much attention [1–13]. It is known that, when an acoustic wave passes through a semiconductor, it may interact with various elementary excitations. In such an interaction the acoustic wave may lose or gain energy under certain circumstances. The latter is known as amplification and the former as attenuation of the acoustic wave. This idea of acoustic wave amplification was theoretically predicted in 1956 by Tolpygo and Uritskii [1] and by Weinreich [2] and experimentally observed for CdS by Hutson *et al* [3] and for n-Ge by Pomerantz [4]. Since then there has been much work carried out, both theoretical and experimental.

The study of this phenomenon in a superlattice (SL) has also attracted attention. Shmelev and Zung [14] have calculated the absorption coefficient and renormalization of the short-wave sound velocity in a SL. Azizyan [15] also calculated the absorption coefficient in a quantized electric field. The possibility of hypersound amplification in a quantized electric field was investigated in [16, 17] and in a non-quantized electric field in [18]. In this paper, we consider the amplification of hypersound in a SL in the presence of both DC and AC electric fields which hitherto has not been considered. The DC electric field is non-quantized, i.e. $eE_0 d \ll 2\Delta$ (d is the period of the SL, 2Δ is the width of the lowest-energy miniband and e is the electron charge). The acoustic wave will be considered in the short-wave region of $ql \gg l$ (q is the acoustic wavenumber and l is the mean free path of an electron). Such an acoustic wave could be considered as a flow of monochromatic phonons (of frequency ω_q).

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0953-8984/94/193479+07$19.50

2. Theory

This problem will be solved in the quasi-classical case, i.e. $2\Delta \gg \tau^{-1}(\hbar = 1)$. The applied electric field $E(t) = E_0 + E_1\cos(\omega t)$ is directed along the SL axis (z axis). The sound absorption coefficient is determined by the expression [19]

$$\Gamma = \frac{|\Lambda|^2 q^2}{4\pi^2 \rho s \omega_q} \int [f(\varepsilon_p) - f(\varepsilon_{p+q})]\delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q)\,\mathrm{d}^3 p \tag{1}$$

where Λ is the constant of deformation potential, ρ is the density of the sample (SL), s is the velocity of sound, $f(\varepsilon_p)$ is the distribution function and p is the momentum of the electrons. To determine the distribution function in the presence of the external field $E(t)$, we solve the Boltzmann equation in the τ-approximation and assume further that τ is constant. The kinetic equation is given by

$$\frac{\partial f(p,t)}{\partial t} + e[E_0 + E_1\cos(\omega t)]\frac{\partial f(p,t)}{\partial p} = -\frac{1}{\tau}[f(p,t) - f_0(p)] \tag{2}$$

and its solution as

$$f(p,t) = \int_0^\infty \exp\left(-\frac{t'}{\tau}\right)\frac{\mathrm{d}t'}{\tau} f_0\left[p - \left(eE_0 t' + \frac{eE_1}{\omega}\{\sin(\omega t) - \sin[\omega(t-t')]\}\right)\right]. \tag{3}$$

Performing the transformation $p \to p - p'$ where

$$p' = eE_0 t' + (eE_1/\omega)\{\sin(\omega t) - \sin[\omega(t-t')]\}$$

and then taking into consideration the energy of SL in the lowest miniband, which is given by

$$\varepsilon(p) = p_\perp^2/2m + \Delta[1 - \cos(p_z d)] \tag{4}$$

where $p_\perp$ and p_z are the quasi-momentum components across and along the SL axis respectively and m is the transverse effective electron mass (in the x–y plane), the equilibrium distribution function is given by

$$f_0(p) = \frac{\pi d n}{m T I_0(\Delta/T)}\exp\left(-\frac{\varepsilon(p)}{T}\right). \tag{5}$$

In equation (5), T is the temperature in units of energy, n is the electron concentration and $I_0(x)$ is the modified Bessel function. We solve equation (1) for a non-degenerate electron gas and obtain for the absorption coefficient of sound

$$\begin{aligned}\Gamma = {} & \frac{(|\Lambda|^2 q^2) n \Theta(1-b^2)}{2\rho s \omega_q \Delta \sin(qd/2\sqrt{1-b^2})}\int_0^\infty \exp\left(-\frac{t'}{\tau}\right)\frac{\mathrm{d}t'}{\tau} \\ & \times \left\{\sinh\left[\frac{\omega_q}{2T}\cos\left(eE_0 t' + \frac{eE_1}{\omega}\{\sin(\omega t) - \sin[\omega(t-t')]\}\right)\right]\right. \\ & \times\cosh\left[\frac{\Delta}{T}\cos(\tfrac{1}{2}qd)\cos\left(eE_0 t' + \frac{eE_1}{\omega}\{\sin(\omega t) - \sin[\omega(t-t')]\}\right)\sqrt{1-b^2}\right] \\ & - \frac{\Delta}{T}\sqrt{1-b^2}\sin\left(eE_0 t' + \frac{eE_1}{\omega}\{\sin(\omega t) - \sin[\omega(t-t')]\}\right) \\ & \times \sin(\tfrac{1}{2}qd)\sinh\left[\frac{\Delta}{T}\cos(\tfrac{1}{2}qd)\cos\left(eE_0 t' + \frac{eE_1}{\omega}\{\sin(\omega t)\right.\right. \\ & \left.\left.\left. - \sin[\omega(t-t')]\}\right)\sqrt{1-b^2}\right]\right\}\end{aligned} \tag{6}$$

where Θ is the Heaviside step function and $b = \omega_q/[2\Delta\sin(\tfrac{1}{2}qd)]$.

3. Results and discussion

We obtain the solution of equation (6) by making the approximation $T \gg \Delta$, ω_q and then integrating and averaging over the period of the AC electric field. The resulting solution is given by

$$\Gamma = \Gamma_0 \sum_{k=-\infty}^{\infty} \frac{J_k^2(z)}{1+(k\omega\tau+z_c)^2}\left[1-\left(\Delta^2(1-b^2)\sin(qd)\sum_{k=-\infty}^{\infty}\frac{J_k^2(z')(k\omega\tau+2z_c)}{1+(k\omega\tau+2z_c)^2}\right)\right.$$
$$\left.\times\left(2T\omega_q\sum_{k=-\infty}^{\infty}\frac{J_k^2(z)}{1+(k\omega\tau+z_0)^2}\right)^{-1}\right] \tag{7}$$

where

$$\Gamma_0 = \frac{(|\Lambda|^2q^2)n\Theta(1-b^2)}{2\rho s\omega_q\Delta\sin(\frac{1}{2}qd)\sqrt{1-b^2}}\frac{\omega_q}{2T}$$

$$z = \frac{eE_1d}{\omega} \qquad z' = 2z \qquad z_c = eE_0d\tau$$

and $J_k(x)$ is the Bessel function.

The following particular cases of equation (7) will be considered.

(i) In the absence of an external field, i.e. at $z = 0$, $z_c = 0$, we obtain from equation (7) the absorption coefficient of phonons [14] given by

$$\Gamma_0 = \frac{(|\Lambda|^2q^2)n\Theta(1-b^2)}{2\rho s\omega_q\Delta\sin(\frac{1}{2}qd)\sqrt{1-b^2}}\frac{\omega_q}{2T}. \tag{8}$$

It is worthwhile to note the appearance of a 'transparency window' when $\omega_q \gg 2\Delta\sin(\frac{1}{2}qd)$; in this situation, $\Gamma_0 = 0$. This is a consequence of conservation laws.

(ii) At $z_c = 0$ from equation (7) we obtain

$$\Gamma = \Gamma_0 \sum_{k=-\infty}^{\infty} \frac{J_k^2(z)}{1+(k\omega\tau+z_c)^2} \tag{9}$$

and, when $\omega \gg q\bar{v}$ and $\omega \gg \omega_q$ ($\bar{v}$ is the characteristic velocity of electrons), we can maintain only $k = 0$ in equation (9), thus obtaining

$$\Gamma = \Gamma_0 J_0^2(z). \tag{10}$$

In this case the coefficient of sound absorption depends on the amplitude of AC electric field in an oscillatory manner. In the asymptotic limit $z \gg 1$, equation (10) gives

$$\Gamma = \text{constant} \times \cos^2\left(\frac{eE_1d}{\omega} - \frac{\pi}{4}\right). \tag{11}$$

It is interesting to observe that equation (11) is similar to that for the coefficient of sound absorption in the quantized electric field [15]. This analogy is probably not accidental since in a quantized electric field electrons undergo Stark oscillations, while in our case electrons in an AC field undergo harmonic oscillations.

(iii) When $z = 0$, we obtain

$$\Gamma = \frac{\Gamma_0(q)}{1 + (eE_0 d\tau)^2} \left(1 - \frac{\Delta^2}{\omega_q T} (1 - b^2) \sin(qd) eE_0 d\tau \frac{1 + (eE_0 d\tau)^2}{1 + (2eE_0 d\tau)^2} \right) \quad (12)$$

which agrees with the result in [18].

From equation (12) we noted that, when $eE_0 d\tau \ll 1$, i.e. in a linear approximation to E_0,

$$\Gamma = \Gamma_0 \left(1 - \frac{\Delta}{\omega_q T} (1 - b^2) eE_0 d\tau \sin(qd) \right) \quad (13)$$

Making use of equation (13) we can find the field strength

$$E_0 > E_0^{\mathrm{SL}} = \frac{\omega_q T}{\Delta^2 ed\tau \sin(qd)} \quad (14)$$

where the sound absorption switches over to amplification, i.e. Γ becomes negative. The value E_0^{SL} has the sense of a threshold field. With $T = 300$ K, $\Delta = 0.1$ eV, $d = 10^{-8}$ cm, $S = 5 \times 10^5$ cm s^{-1}, $\tau = 10^{12}$ s and $\omega_q = 10^{10}$ s^{-1} the threshold value $E_0^{\mathrm{SL}} = 18.7$ V cm^{-1}. As indicated in [18], this value is smaller than that of a homogeneous semiconductor (bulk material) where $E_0^{\mathrm{hom}} = 60$ V cm^{-1} (at the same value of τ and with effective mass $m = 0.2m_e$).

Furthermore, when $eE_0 d\tau \gg 1$, we obtain from equation (12)

$$E_0^{\mathrm{SL}} = \frac{4\omega_q T}{\Delta^2 ed\tau \sin(qd)} \quad (15)$$

which is four times that in equation (14). Thus, using the same parameters as indicated above, we obtain $E_0^{\mathrm{SL}} = 74.8$ V cm^{-1}.

In fact, for $eE_0 d\tau \gg 1$, an analogous result can be obtained from figures 1 and 2. The solid curve represents the response in the absence of an AC field. This curve cuts the z_c axis at approximately 0.34. Equating this value to z_c and making E_0^{SL} the subject, we obtain

$$E_0^{\mathrm{SL}} = \frac{0.34\hbar}{ed\tau}. \quad (16)$$

(Note that $\hbar = 1$ everywhere but for numerical evaluation we have indicated it.) This then gives the value of E_0^{SL} to be about 74.7 V cm^{-1} which agrees very well with the numerical calculations.

(iv) Finally, when $\omega\tau \ll 1$, we obtain from equation (7) after a lengthy manipulation

$$\Gamma = \Gamma_0(q) \left(\frac{\frac{1}{4}\{[1 + (\omega_s + \beta)^2]^{1/2} + [1 + (\omega_s - \beta)^2]^{1/2}\}^2 - \omega_2}{[1 + (\omega_s + \beta)^2][1 + (\omega_s - \beta)^2]} \right)^{1/2}$$
$$\times \left(1 - \frac{\Delta^2}{2\omega_q T} (1 - b^2) \sin(qd) (\tfrac{1}{4} [\![\{[1 + (\omega_s' + \beta')^2]^{1/2} + [1 + (\omega_s' - \beta')^2]^{1/2}\}^2 \right.$$
$$- \beta'^2 - 1]\!] / [1 + (\omega_s' + \beta')^2][1 + (\omega_s' - \beta')^2])^{1/2} ([\![\tfrac{1}{4}\{[1 + (\omega_s + \beta^2)^{1/2}]$$
$$\left. + [1 + (\omega_s - \beta)^2]^{1/2}\}^2 - \omega_s^2]\!] / [1 + (\omega_s + \beta)^2][1 + (\omega_s - \beta)^2])^{-1/2} \right) \quad (17)$$

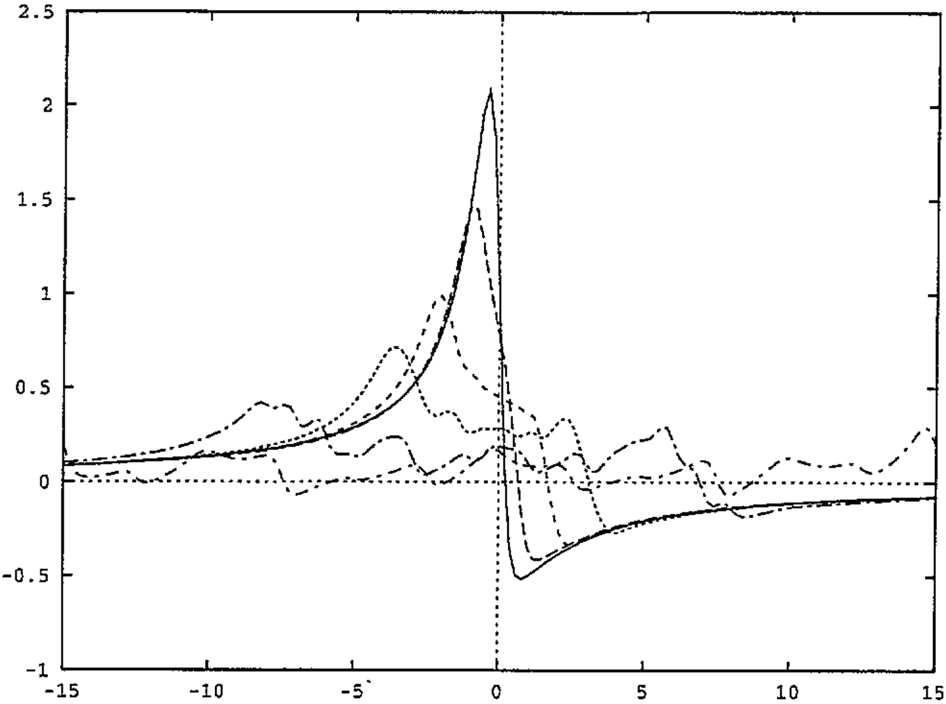

Figure 1. Dependence of Γ/Γ_0 on Z_c: ——, $z = 0$; — — —, $z = 4$, $\omega\tau = 0.2$; - - -, $z = 4$, $\omega\tau = 0.5$; ----, $z = 4$, $\omega\tau = 0.9$; — · · —, $z = 4$, $\omega\tau = 2$; — · —, $z = 4$, $\omega\tau = 5$.

where $\omega_s = eE_0d\tau$, $\omega_s' = 2\omega_s$, $\beta = eE_1d\tau$ and $\beta' = 2\beta$.

The dependence of Γ on E_0 for a given E_1 is presented graphically in figure 1.

The figure shows the absorption coefficient Γ against constant electric field E_0. The peak values decrease with increasing values of $\omega\tau$ for $\omega\tau \ll 1$. However, at $\omega\tau = 0.9$, some slight oscillation is observed and, for $\omega\tau \gg 1$, e.g. $\omega\tau = 5$, the curve oscillates. We also observe a shift in the peaks for different values of $\omega\tau(\ll 1)$. This can be explained with the help of equation (17). Whenever $\omega_s = \beta$, i.e. $E_0 = E_1$, we expect a peak in that neighbourhood, since $\beta = z\omega\tau$, so that, as $\omega\tau$ increases, the peaks shift towards larger E_0. Figure 2 indicates a decrease and shift in the peak values of different z for a given $\omega\tau(\ll 1)$.

It is also interesting to note that for $\omega\tau \ll 1$ the threshold field E_0^{SL} can be obtained from the figures. Figure 1 reveals that the threshold field E_0^{SL} increases with increasing $\omega\tau$ for a given E_1. Similarly figure 2 indicates that E_0^{SL} increases with increasing E_1 for a given $\omega\tau$. Hence, to obtain an optimal threshold field E_0^{SL}, it is necessary to regulate both E_1 and $\omega\tau$.

As indicated in [18], the non-linear behaviour of Γ with E_0 may permit the use of a SL as a hypersound generator in a way similar to the long-wave sound generator operating on a homogeneous semiconductor [20, 21].

4. Conclusion

The propagation of hypersound in a SL in the presence of DC and AC electric fields has

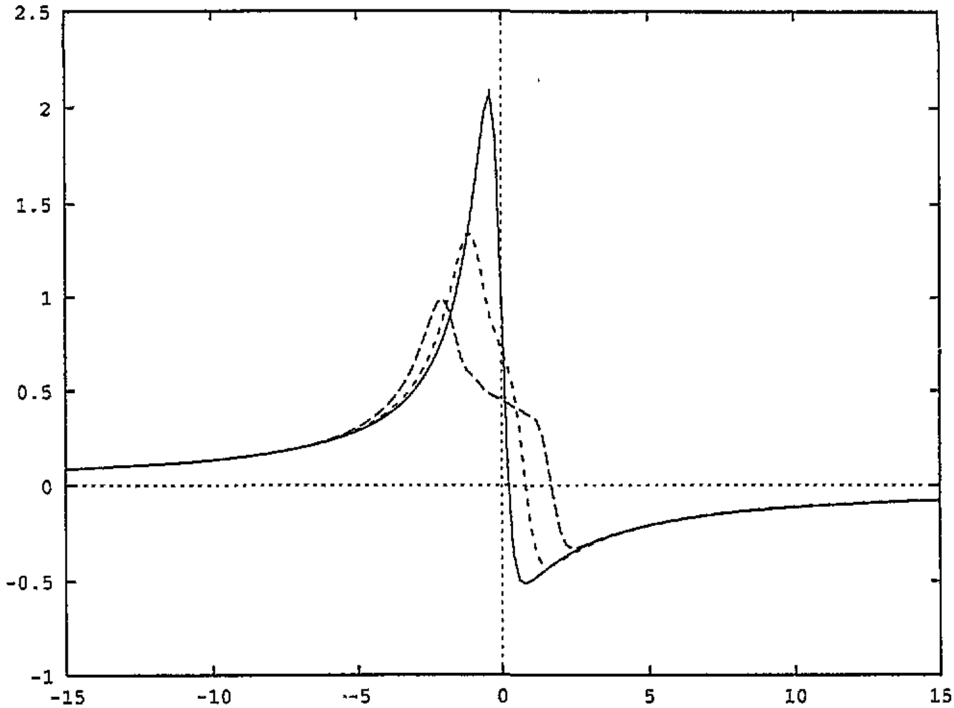

Figure 2. Dependence of Γ/Γ_0 on Z_c: ——, $z = 0$; - - -, $\omega\tau = 0.5$, $z = 2$; — — —, $\omega\tau = 0.5$, $z = 4$.

been studied theoretically. Analytical expressions have been obtained for the absorption coefficient Γ under different conditions. It is shown that the dependence of Γ on E_0 is quite different from that of a homogeneous semiconductor. In particular, when an AC field is absent or weak ($\omega\tau \ll 1$), we observe a non-linear dependence of Γ on E_0. When $\omega\tau \gg 1$ the dependence of Γ on E_0 tends to be oscillatory in form.

Acknowledgments

The authors would like to thank Professor Abudus Salam, the International Atomic Energy Agency and UNESCO for their fine hospitality at the International Centre for Theoretical Physics, Trieste. They would also like to thank the Swedish Agency for Research Cooperation with Developing Countries (SAREC) for financial support during their visit to the ICTP under the Associateship Scheme as well as Dr Abdul Qadeer for his tremendous help in assisting with the computation of the results.

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